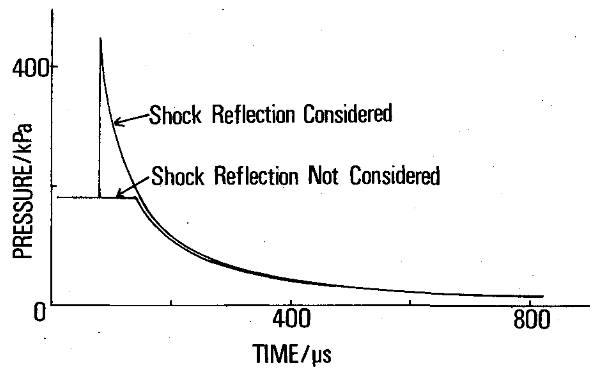

Fig. 3 Pressure changes along particle path where shock reflection has and has not been considered (zero in time corresponds to rupture of secondary diaphragm).

theoretical profiles to fit their experimental traces. This time shift can be attributed to the finite opening time of the diaphragm and a detailed investigation of this has been performed by Hall et al.[10] Experimental pressure traces at 100 and 455 mm downstream from the secondary diaphragm require the same time shifts to fit the corresponding theoretical profiles. This suggests that the model is least reliable during the earliest stages of expansion—the region where neglect of reflection gives completely incorrect results (see Fig. 3).

It is clear that our analysis adequately describes the important aspects of the flow, but it is appropriate to consider possible sources of error in the above treatment. The assumption of shock reflection from a plane surface does not allow for bulging of the secondary diaphragm caused by the pressure differential across it prior to rupture. It was found that aluminum foil diaphragms subjected to test gas pressures of about 40 kPa stretched to form a dome about 10 mm high. Assuming reflection to take place from a plane surface at the most stretched point of the diaphragm (i.e., 10 mm downstream) we found little effect on the computed pressure profiles. Errors in measuring Δt had negligible effect.

In using the double-diaphragm shock-tube technique it is necessary to know the period over which ideal flow occurs. The end of ideal flow may be caused by the arrival of the contact surface or reflected rarefaction head (generated by the driver expansion) or by the arrival of secondary-diaphragm fragments. The reflected rarefaction wave can be retarded so that it does not interfere with the flow by using a low-sound-speed driver gas such as nitrogen rather than the more usual hydrogen or helium. The arrival of diaphragm fragments, observed by light scattering (see Fig. 2) often coincides with the arrival of the contact surface.

Bursting times of secondary diaphragms are very dependent on the nature and thickness of the diaphragm material. For 0.009 mm thick aluminum foil the reflected shock had decayed completely before reaching the upstream window. Cellulose acetate film 0.17 mm thick gave reflected shock waves of long persistence. The apparent failure of others to observe shock reflection from the secondary diaphragm can now be explained by the occurrence of shock decay before arrival at the pressure measuring station.

The importance of taking into account shock reflection can best be seen in Fig. 3. The pressure change along the particle path reaching the downstream position at a given time is plotted for cases where shock reflection is and is not considered. It can be seen that the major differences occur immediately behind the reflected shock front and that for most of the expansion the rate of change of pressure is roughly the same.

We conclude, therefore, that shock reflection does occur. Agreement between pressure traces measured downstream from the secondary diaphragm and theoretical profiles in which shock reflection has not been considered cannot be taken as evidence that shock reflection is insignificant. Pressure measurements upstream from the secondary diaphragm need to be made. The omission of shock reflection from analyses can lead to serious inaccuracies in the earliest parts of the expansion where significant chemical or physical changes can occur at the much higher temperatures and pressures behind the reflected shock.

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AIAA 81-4067

Pressure-Coupled Response of Composite Solid Propellants

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Introduction

With the advent of standardized stability prediction codes[1] acquisition of adequate response function data has become a major impediment to routine inclusion of realistic linear stability analyses in the motor design process. This is a particularly difficult problem for motors destined for tactical applications.[2] A practical solution path is to employ results from combustion models to provide a framework for correlating data. This path has been pursued for composite propellants by employing both homogeneous and heterogeneous propellant models. (Reference 3 has recently reviewed response function theories that account for heterogeneity.) Limited success has been achieved as both models can correlate the available T-burner data about

Received April 14, 1980; revision received Sept. 16, 1980.

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equally well. However, the heterogeneous models, built largely upon intuitive postulates, exhibit multiple resonance characteristics for the response functions of some propellant formulations. Since the homogeneous models exhibit single resonance behavior and multiple resonance characteristics are not evident in available T-burner data, the heterogeneous models have been viewed with suspicion. However, recent (and very limited) data obtained via microwave interferometry[4] show definite multiple resonance characteristics. The importance of response function behavior to both stability prediction and the elimination of instability[5] makes the existence of multiple resonances particularly relevant to solid rocket design. The objective of this work is to show, for a limited formulation domain, that multiresonance characteristics are probable for multimodal composite propellants. Attention is focused herein on the low and intermediate frequency regimes where the reactive zone can be presumed to show quasisteady behavior.

Analysis

In steady-state statistical combustion models[6] mean values are computed as spatial averages at fixed time. If the heterogeneous structure is random, the ergodic theorem suggests that an equivalent procedure would be to compute temporal averages at a fixed location on the burning surface. Thus, a model in which propellant layers are parallel to the burning surface is a plausible idealization. However, to be realistic the burning surface must move relative to the layers and the layers must be compatible with pseudopropellant properties. That is, the layers should be the thickness of an oxidizer particle and randomly ordered and should possess the thermophysical properties of the appropriate pseudopropellant. Data correlations for additive-free AP/hydrocarbon composite propellants show that acceptable results occur when the O/F ratio is the same for all pseudopropellants.[7] Accordingly, as a first approximation, the thermophysical properties of all pseudopropellants are equivalent. Thus, a plausible layer model embodies elements of the models of both Williams and Lengelle[8] and Cohen and Bowyer[9] but neglects thermo-property variations. That is, the condensed phase is thermally homogeneous; differences among the pseudopropellant layers arise from their intrinsically different rates, exponents, temperature sensitivities, etc.

Consider a bimodal propellant composed of coarse oxidizer particles whose weight mean diameter is much larger than either the Michelson length ($d_{\mathrm{ox},c} \gg \delta_c$) [Michelson length δ = (thermal diffusivity)/(burning rate)] or the weight mean diameter of the fine oxidizer mode ($d_{\mathrm{ox},c} \gg d_{\mathrm{ox},f}$). The number of pseudopropellant layers of coarse material in a mass m of this propellant is (mass of pseudopropellant in m/mass of a pseudopropellant layer)

$$N_c = m_c / (A d_{\mathrm{ox},c} \rho) \tag{1}$$

and the number of fine pseudopropellant layers in this mass is

$$N_f = m_f / (A d_{\mathrm{ox},c} \rho) \tag{2}$$

where A is the cross-sectional area and ρ is the density. Suppose that the combustion process has just consumed a coarse pseudopropellant layer. What is the probability of the next layer being fine pseudopropellant? This is the problem of drawing a black ball from an urn filled with a very large number of black and white balls. Therefore, the probability of the next layer being fine pseudopropellant is

$$P_f = N_f / (N_c + N_f) = [1 + m_c d_{\mathrm{ox},f} / (m_f d_{\mathrm{ox},c})]^{-1} \tag{3}$$

Introduce the diameter ratio $\beta = d_{\mathrm{ox},c} / d_{\mathrm{ox},f}$ and the oxidizer fraction $x_f = m_f / (m_f + m_c)$. Then ask what is the probability of finding M adjacent layers of fine pseudopropellant.

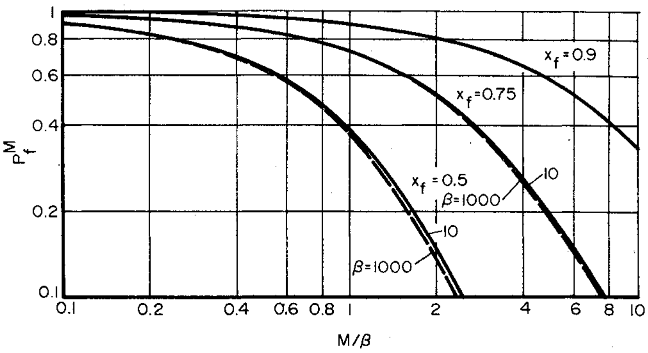

Fig. 1 Probability of fine pseudopropellant strata with thickness *M/β*.

(Adjacent layers of the same pseudopropellant will be referred to as a pseudopropellant strata.) This is equivalent to drawing M black balls in M tries from the urn. This probability is

$$P_f^M = (P_f)^M = [1 + (1 - x_f)/(x_f \beta)]^{-M} \tag{4}$$

Figure 1 presents P_f^M vs M/β with x_f and β as parameters. The figure shows that P_f^M is sensibly independent of diameter ratio in M/β coordinates. Moreover, for $x_f > 0.75$ the probability of finding a strata of fine pseudopropellant equivalent in thickness to a coarse pseudopropellant layer ($M/\beta = 1$) is better than 7/10. Thus, for $x_f > 0.75$, the time mean characteristics of the layered structure will be dominated by strata of coarse and fine pseudopropellant with thicknesses much greater than their Michelson lengths. Consequently, in this case the combustion phenomena in each strata will be only slightly dependent upon coarse-to-fine and fine-to-coarse transients. In other words, each strata will behave essentially as a single independent propellant. Therefore, the time mean burning rate of the layered propellant becomes

$$\langle r \rangle^{-1} \simeq x_f r_f^{-1} + x_c r_c^{-1} \tag{5}$$

and the time mean response function $\langle R_p \rangle$ is approximately

$$\langle R_P \rangle \simeq (x_f R_{p,f} / r_f + x_c R_{p,c} / r_c) \langle r \rangle \tag{6}$$

It is clear that these approximations improve as $d_{\mathrm{ox},c}/\delta_c$ and x_f increase.

Equation (6) shows that the time mean response function for a randomly layered pseudopropellant medium in which $d_{\mathrm{ox},c} \gg \delta_c$ is composed of response functions for the pseudopropellants comprising that medium. Since these response functions will generally differ because their rates, exponents, and temperature sensitivities differ, and will possess at least single resonance characteristics, the time mean response function $\langle R_p \rangle$ will generally exhibit multiresonance characteristics. Moreover, note that neither $d_{\mathrm{ox},c}$ nor $d_{\mathrm{ox},f}$ appears explicitly as a characteristic dimension. Rather, the Michelson lengths are the characteristic dimensions. Therefore, in this limiting situation "layer frequency phenomena" do not appear unless they are implicit in $R_{p,c}$ and $R_{p,f}$. This suggests that the layer frequency concept has, at most, restricted domains of application.

The analysis herein pertains to a limited formulational domain. Numerical modeling of the general situation is currently underway. These results will be reported when they become available.

Acknowledgments

Research was sponsored by the Air Force Office of Scientific Research, United States Air Force, under Grant 79-0022.

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AIAA 81-4068

Exploratory Experiments on Acoustic Oscillations Driven by Periodic Vortex Shedding

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Nomenclature

a = sound speed
f = frequency
ℓ = characteristic dimension
L = cavity length
$\bar{M}$ = average Mach number
n = acoustic mode number
S = Strouhal number
u = average chamber velocity

Subscripts

a = acoustic
c = critical
s = value associated with vortex shedding

ANALYSIS of pressure data from a number of segmented solid propellant rocket motors has shown the presence of low-amplitude pressure oscillations. However, when the acoustic stability analyses of Culick[1] and Cantrell and Hart[2] are applied to motors, they are all predicted to be stable. At first one might suspect the accuracy of the propellant combustion response values used in these analyses. Review of these data, however, shows they are relatively repeatable. Furthermore, their values are often less than unity and, hence, are well below the values required to overcome the nozzle and flow turning losses. Inclusion of particle damping only increases the response required to generate pressure oscillations spontaneously. Thus there is little reason to suspect that errors in the propellant properties can account for the observed oscillations. Hence, it appears that a significant source of acoustic energy has been omitted from these analyses.

Several years ago, Flandro and Jacobs[3] suggested that periodic vortex shedding could interact with the chamber acoustics to generate pressure oscillations. A review of this coupling has been prepared recently by Flandro.[4] Also, Culick and Magiawala[5] have described experimental results on acoustic interactions with vortices using a blower-driven tube with no flow restrictions at the downstream end.

The experiments reported in this paper studied this mechanism under conditions which simulate segmented rocket motor acoustics more closely. In particular, the objectives were to study the effect of flow across protruding inhibitors which often exist in this type of rocket motor chamber, and to determine the conditions which produced significant coupling between the flow and the acoustics.

The essential features of the apparatus are shown in Fig. 1. The flow cavity consisted of carbon steel tubing with an internal diameter of 3.8 cm and a length of approximately 106 cm. Nitrogen entered the chamber through a sonic choke and exhausted through a second sonic choke. Thus the chamber was acoustically isolated from the surrounding environment. Strain gage-type transducers monitored the pressure upstream of both chokes. In addition, a Kistler transducer was located at the head end of the simulated chamber to monitor the acoustic pressure.

The tubing was sectioned at the midpoint so that the spacing between the two restrictors could be varied. These restrictors had a port of 1.9 cm. Provision was made for inserting a hot wire anemometer midway between the two restrictors.

The shedding frequency of periodic vortex flows can be characterized by the Strouhal number

$$S = f_s \ell / u \tag{1}$$

where ℓ is a characteristic length and f_s is the shedding frequency. If the shedding frequency equals the frequency of one of the acoustic modes, one suspects significant driving of acoustic oscillations could be generated. Since the acoustic mode frequency is

$$f_a = na/2L \tag{2}$$

one would expect conditions of high acoustic level to be characterized by

$$S_c = n\ell / 2\bar{M}L \tag{3}$$

This simple concept was tested by varying the restrictor spacing ℓ and determining the conditions required for maximum acoustic pressure.

Initial experiments were run with all restrictors removed. The amplitude spectrum showed dominant peaks corresponding to the first six axial modes of the apparatus. Next, a series of tests was conducted with two restrictors located at the $L/2$ position. The hot wire anemometer was located approximately midway between the two restrictors, and at the diameter of the restrictor orifice. The initial spacing between the two restrictors was 1.42 cm. Figure 2 shows the frequency spectra obtained from the Kistler transducer and the hot wire anemometer for a Mach number of 0.042 through the restrictor orifices. These data show no amplification of any of the acoustic frequencies. In fact, there appeared to be some damping of the first few modes. Next, the spacing between the restrictors was increased in small increments to a maximum value of 2.54 cm. The results obtained at 2.0 cm,

Received May 16, 1980; revision received Aug. 25, 1980.

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